INFLUENCE OF THE MAGNUS FORCE ON THE MOTION OF A SPHERICAL SOLID WITH A LARGE ANGULAR VELOCITY

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UDC 532.529

The influence of the initial angular velocity imparted by an electric motor to a spherical solid on its deviation from the vertical in fall is investigated experimentally. Values of the coefficient C_M in the formula for the Magnus force at which the trajectories of sphere motion are in agreement with the experimental data are found by calculation. It is established that as the Reynolds number Re_ω grows the coefficient C_M decreases; with $Re_\omega \sim 3 \cdot 10^4$ C_M is 10% of the quantity C_M^0 found by Rubinov and Keller for small Reynolds numbers.

In rotation of a solid in a gas flow it also experiences, in addition to the aerodynamic drag force F_a, the Magnus lateral (lifting) force F_M. In [1] the expression for this force for $Re_v \to 0$, $Re_\omega \to 0$ and a spherical form of the solid was obtained:

$$F_M = C_M \rho \left(\frac{\delta}{2} \right)^3 \omega \times V, \quad C_M = C_M^0 = \pi. \tag{1}$$

As $Re_v \to \infty$, $Re_\omega \to \infty$ the asymptotic value of the coefficient $C_M^\infty = 8\pi/3$ is obtained (see [2]). Experimental and theoretical investigations of the dynamics of rotating spherical solids in [3-6] with substantial Reynolds numbers ($Re_v \gg 1$, $Re_\omega \gg 1$), however, have shown that the value of the coefficient in Eq. (1) is less than C_M^0.

The above works are analyzed in [3] but the graphical form for representation of C_M as a function of $\Gamma = \delta\omega/(2V_r)$ is such that it is extremely difficult to establish some law. The survey [7] gives values of the coefficient C_M from various works with an indication of just the values of Re_v for which the experiments were performed:

$$C_M = \begin{cases} 2,2 - 3,7; & Re_v = 10 - 30 & [8], \\ \pi(0,2 - 0,05); & Re_v = 550 - 1600 & [3], \\ \pi(0,8 - 1); & Re_v = 2000 - 6000 & [4], \\ 0,35 & & [5]. \end{cases}$$

We note that in [5] the value of C_M is obtained for very large particles. From analyzing the data presented it follows that as Re_v grows C_M behaves nonmonotonically.

To determine C_M, elements of the trajectory of a solid spherical particle in a tube were measured in [4], and the particle, in addition to the Magnus force, also experienced the Saffman lifting force due to a gas velocity gradient in the wall region. Two coefficients in the expressions for each of these forces were simultaneously determined. In [3] the angular velocity and the displacement of the particle due to the action of the Magnus force were measured after impact against a rough surface, which could introduce a considerable error into the experimental data.

With the aim of determining C_M for large Reynolds numbers Re_ω experimental investigations of the laws of motion of a rotating freely falling spherical solid were performed at the Institute of Problems of Energy Saving of the Academy of Sciences of the Ukraine. The measurements were performed on the stand whose diagram is
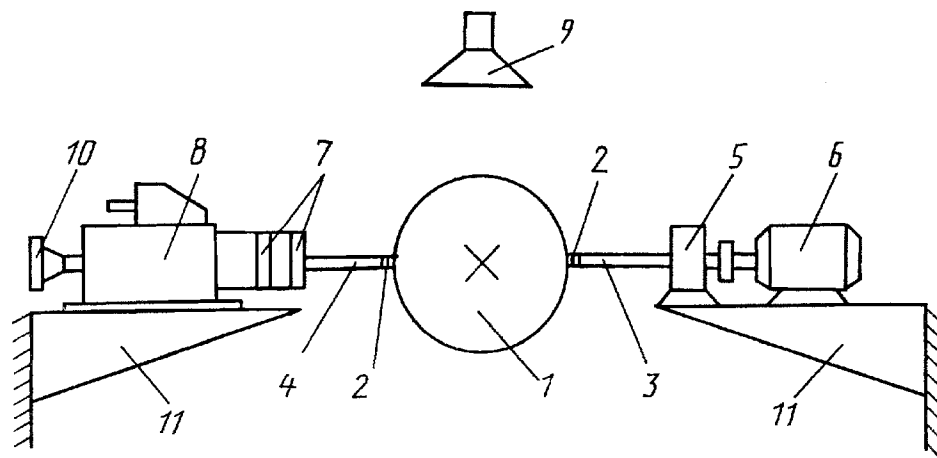

Fig. 1. Diagram of the experimental installation.

given in Fig. 1. Celluloid sphere 1 with diameter $\delta = 37.8$ mm was positioned between rubber tips 2 glued to the ends of shafts 3, 4. Shaft 3 via reducer 5, which made it possible to vary the number of revolutions, was put in rotation by electric motor 6. The torque was transmitted via sphere 1 to shaft 4, pressed into bearing 7 of the electromagnet armature 8. As a result shafts 3, 4 and sphere 1 rotated with the same angular velocity ω_0. The rotational speed was checked by stroboscopic tachometer 9 on a label applied to the sphere. The force necessary to hold the rotating sphere between the tips was created by a spring loading the armature and located inside the electromagnet. The compression of the spring was adjusted the by screw 10. The electromagnet and the motor were installed on arms 11 located at some height H from the floor.

At the instant the electromagnet was started its armature drew in and the rotating sphere fell down. In the experiments we measured the abscissa of the fall point x_e reckoned from the vertical passing through the center of the sphere located at the initial instant between the rubber tips. At each rotational speed a series of 20-25 experiments was performed; the results of each series were averaged. Table 1 gives the mean values of x_e and the root-mean-square deviations δ_x in three series of experiments for H = 2.1 m.

From Table 1 it can be seen that as the initial angular velocity of the sphere ω_0 grows the abscissa of the fall point x_e increases. The value of x_e can also be determined from calculation. It was initially established by calculation that the force factor associated with the effect of additional masses and the Basset force have practically no effect on the trajectory of motion of the sphere in question. This enables us to state that the determining factor for deviation of the falling sphere from the vertical is the Magnus force F_M,,whose value depends on the value of the coefficient C_M.

To calculate the trajectory of motion of the solid, the following system of equations was used:

$$m\,du/dt = -au + bv\omega, \tag{2}$$

$$m\,dv/dt = -av - bu\omega + gm, \tag{3}$$

$$I\,d\omega/dt = (C_\omega/64)\,\rho\delta^5\omega^2, \tag{4}$$

where $a = (\pi\delta^2/8)\rho V C_R$; $b = C_M\rho\delta^3/8$; $I = m\delta^2/6$.

The aerodynamic drag coefficient was calculated by Kravtsov's formula [9]

$$C_R = \begin{cases} 0{,}32 + 4{,}3/\sqrt{Re_v} + 24/Re_v, & Re_v < 800; \\ 0{,}44, & Re_v > 800. \end{cases} \tag{5}$$

In [5] on the basis of analyzing experimental data of various authors it is pointed out that rotation of a spherical body has an insignificant effect on the aerodynamic drag coefficient C_R.

In [10] based on a numerical solution of the problem of flow around a particle with $Re_\omega = 80\text{-}4000$ the formula for the coefficient of the moment of resistance of the gas acting on a rotating spherical body is obtained:

Table 1

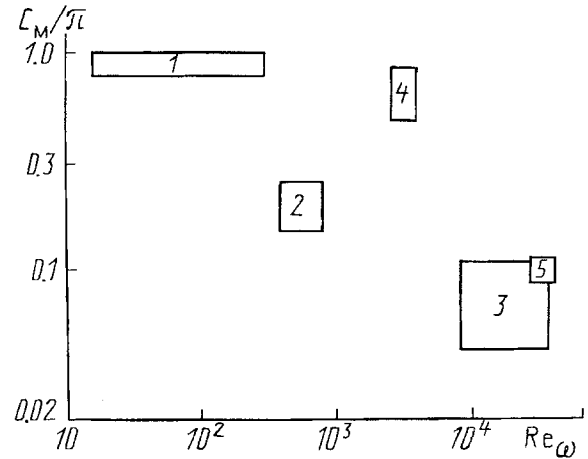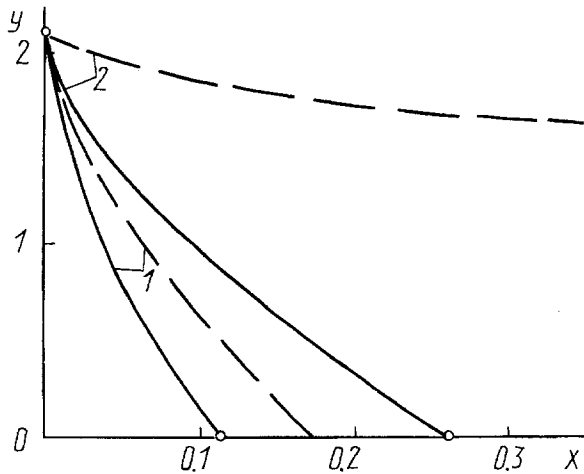| No. of the series of experiments	ω_0, sec^{-1}	x_e, m	σ_x, m
1	25.6	0.116	0.015
2	35.9	0.167	0.014
3	386.4	0.257	0.024

Fig. 2. Trajectories of motion of a spherical body: 1) $\omega_0 = 25.6$ sec^{-1}; 2) $\omega_0 = 386.4$ sec^{-1}. Dashed lines are the results of calculation with $C_M = C_M^0 \cdot x$, y, m.

Fig. 3. Coefficient C_M in the expression for the Magnus force vs. Reynolds number Re_ω: 1) according to the data of [4]; 2) [3]; 3) [6]; 4, 5) determined in the present work, 4) according to series of experiments 1 and 2 (see Table 1), 5) according to series of experiments 3.

$$C_\omega = 12{,}9/\sqrt{Re_\omega} + 128{,}4/Re_\omega, \tag{6}$$

which also agrees with experimental data for much larger Re_ω [10]. We note that in [4] Eq. (6) is also used, but Re_v is mistakenly substituted for Re_ω.

The system of equations (2)-(4) with the initial conditions $u_0 = v_0 = 0$, $\omega = \omega_0$ was solved numerically by the Runge-Kutta method. In Fig. 2 the trajectories of sphere motion calculated with $C_M = C_M^0$ are shown by the dashed lines. It can be seen that with a large angular velocity of the sphere the Magnus force causing its horizontal displacement is substantially overestimated.

By numerical investigations it was established that (taking into account the measuring error for x_e) the deviation consistent with the experimental x_e is as follows: at $\omega_0 = 25.6$ sec^{-1} $C_M = \pi(0.657 \pm 0.183)$, at $\omega_0 = 386.4$ sec^{-1} $C_M = \pi(0.098 \pm 0.011)$.

We will compare the obtained results with the data of [3, 4, 6] but will represent them as a function of Re_ω rather than as in [3] or in [7]. (The results of [8] are not dealt with here since they have been obtained with a very large error. Besides, the value of Re_ω is not indicated.) Figure 3 shows the results of the comparison. Taking account of the fact that both in the present work and in [3, 4, 6] the measurements and calculations have been performed in specific ranges of Re_ω and the value of C_M has been calculated with some deviations, the regions of variation of the above values are shown as rectangles in Fig. 3. Here rectangle numbers correspond to the data of [4, 3, 6]. It can be seen that in the range of variation of Reynolds numbers Re_ω indicated in Fig. 3 the value of the coefficient C_M in the formula for the Magnus force decreases with increasing Re_ω.

In our experiments as in the experiments of [3-6] we failed to detect a tendency toward an increase in the coefficient C_M with growing Re_ω and its approach to the asymptotic value C_M^∞ indicated above. In calculations it is necessary to take into account that with $Re_\omega \sim 3 \cdot 10^4$ C_M is 10% of the value of C_M^0 found by Rubinov and Keller for small Reynolds numbers.

NOTATION

V, velocity of the center of mass of the spherical solid; u, v, projections of the velocity on the x- and y-axis, respectively; V, modulus of the velocity of the center of mass; g, free fall acceleration; ρ, ν, density and coefficient of kinematic viscosity of the gas; ω, angular velocity of the spherical solid; m, δ, its mass and diameter; $Re_v = V\delta/\nu$, $Re_\omega = \omega\delta^2/\nu$, Reynolds numbers; C_R, C_M, C_ω, coefficients of the aerodynamic drag force F_a, the Magnus force F_M, and the moment of resistance respectively; I, axial moment of inertia of a spherical solid whose mass is distributed over its surface; t, time. Indices: r, relative value of the quantity.

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